GU-RET 2016

GAUHATI UNIVERSITY RESEARCH ELIGIBILITY TEST

MATHEMATICS

Booklet Series : B

Invigilator's Name and Signature	**BOOKLET NO.**	
	OMR SHEET NO.	
	ROLL NO.	

TIME : 2 HOURS 20 MINUTES **TOTAL MARKS : 80**

Number of Pages in this Booklet : **18**

Instructions for Candidates

1. Write your Roll No. and OMR Sheet No. in the boxes provided above.
2. This paper consists of two sections : **Section B** with 50 (fifty) multiple choice questions (MCQ) and **Section C** with 8 (eight) descriptive questions. Each MCQ has 4 (four) answers, out of which ONLY one is correct. You have to darken the circle (on the OMR Sheet) for the correct answer corresponding to the question given in this booklet.

   Example : (A) (B) (C) (D)

   where (C) is the correct answer. No marks will be given for markings made in this booklet. The descriptive questions in **Section C**, MUST be answered in the space provided in this booklet. **No extra pages will be provided in any case.**
3. Use a BLACK ball point pen in your OMR Sheet.
4. Read the instructions given inside this booklet before attempting to answer any questions.
5. DO NOT write your name, roll no, phone no, or anything, or put any marks anywhere in this booklet, otherwise your candidature will be disqualified.
6. If you are found to resort to any kind of unfair means such as carrying extra material other than pen, pencil, watch, eraser, and scale, or copying from somebody or from external material, your candidature will be disqualified.
7. Use of mobile phones, programmable calculators, log tables or any other tables, wearable smart devices such as smart Android watches or objects of similar nature CAN NOT be used inside the examination hall.
8. At the end of the examination, you have to return this booklet and the OMR Sheet back to the invigilator.
9. There is no negative marks for incorrect answer.

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Section B (50 Marks)

1. The value of the Legendre symbol $(2/p)$ is -1. Which of the following is a possible value of P?

   (A) 7
   (B) 17
   (C) 13
   (D) 23

2. The particular integral of
   $$(D^2 - 6D + 9)y = 4e^{3x}$$
   is

   (A) $2xe^{3x}$
   (B) $-2xe^{3x}$
   (C) $2x^2e^{3x}$
   (D) $2e^{3x}$

3. The equation
   $$x^2y\frac{\partial^2 z}{\partial x^2} + xy\frac{\partial^2 z}{\partial x \partial y} - y^2\frac{\partial^2 z}{\partial y^2} = 0$$
   is

   (A) Parabolic when $y = -1/2$
   (B) Parabolic when $y = x$
   (C) Parabolic when $y = -1/4$
   (D) Parabolic when $y = 1/2$

4. The singular solution of $z = px + qy - 2\sqrt{pq}$ is

   (A) $x = -y$
   (B) $x = y$
   (C) $x + y = 1$
   (D) $xy = 1$

5. The Laplace transform of the function $F(t) = \frac{1}{\sqrt{\pi t}}$ is

   (A) $1/s$
   (B) $1/\sqrt{s}$
   (C) s
   (D) $\sqrt{s}$

6. If $F(s) = L(f(t))$, then $L\left(\int_0^t f(x)\,dx\right)$ is equal to

   (A) $F(s)$
   (B) $F(s)/s^2$
   (C) $F(s)/s$
   (D) $-F(s)$

7. Given $f(x) = \begin{cases} 1, |x| < a \\ 1, |x| > a \end{cases}$, then $F\{f(x)\}$

   (A) $\frac{1}{\lambda}2\sin(\lambda a)$
   (B) $\frac{1}{\lambda}2\cos(\lambda a)$
   (C) $\frac{1}{\lambda}\sin(\lambda a)$
   (D) $\frac{1}{\lambda}\cos(\lambda a)$

8. If $f(\infty) = 0$ and $F(\alpha) = F_s(f(x))$, then $F_c(f'(x))$ is equal to

   (A) $-\sqrt{\frac{2}{\pi}}f(0)$
   (B) $\sqrt{\frac{2}{\pi}}f(0)$
   (C) $\alpha F(\alpha)$
   (D) $\alpha F(\alpha) - \sqrt{\frac{2}{\pi}}f(0)$

9. The differential equation
   $$y''(x) - 3y'(x) + 2y(x) = 4\sin x$$
   with initial conditions $y(0) = 1, y'(0) = -2$ represents

   (A) Fredholm integral equation of second kind
   (B) Fredholm integral equation of first kind
   (C) Volterra integral equation of first kind
   (D) Volterra integral equation of second kind

10. The extremal of $\int_0^2 y'^2/x\,dx$ with $y(0) = 0$ and $y(2) = 1$ is

    (A) a parabola
    (B) a circle
    (C) an ellipse
    (D) a straight line

11. The magnitude of the impulse developed by a mass of 200 gm which changes its velocity (in m/s) from $\boldsymbol{v}_1 = 5\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}} + 7\hat{\boldsymbol{k}}$ to $\boldsymbol{v}_2 = 2\hat{\boldsymbol{i}} + 3\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}$ is

(A) 0.9 Ns
(B) 1.8 Ns
(C) 3.6 Ns
(D) 7.2 Ns

12. Which of the following statement is false?

(A) If a particle moves freely in space, the degree of freedom of the system is three.
(B) If a particle moves on the surface of sphere, the degree of freedom of the system is three.
(C) If a particle moves on a circle, the degree of freedom of the system is one.
(D) If the Lagrangian does not depend on time explicitly, the Hamiltonian is constant.

13. For the system with Hamiltonian $H = q_1 p_1 - q_2 p_2 + q_1^2 + q_2^2$, which one of the following is true?

(A) $\dot{q_1} = -q_1$
(B) $\dot{q_2} = q_2$
(C) $\dot{p_1} = (p_1 + 2q_1)$
(D) $\dot{p_2} = p_2 - 2q_2$

14. Suppose U and V are distinct 4-dimensional subspaces of a vector space of dimension 6. The possible dimension of $U \cap V$ is

(A) 2 or 4
(B) 4 or 6
(C) 2 or 5
(D) 2 or 3

15. Let S and T be the linear operators on R^2 defined by $s(x, y) = (0, x)$ and $T(x, y) = (x, 0)$. Then

(A) $TS = 0$, $ST \neq 0$
(B) $TS = 0$, $ST = 0$
(C) $TS \neq 0$, $ST \neq 0$
(D) $TS \neq 0$, $ST = 0$

16. The intersection number ω for the graph $K_{3,3}$ is

(A) 9
(B) 3
(C) 6
(D) None of the above

17. The graph $\overline{K_m} + \overline{K_n}$ is

(A) K_m
(B) K_n
(C) $K_{m,n}$
(D) $\overline{K_{m,n}}$

18. The square of a polygon C_5 is

(A) $\overline{C_5}$
(B) K_5
(C) $\overline{C_5}$
(D) $K_{5,5}$

19. The set

$$A = \left\{0, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}\right\}$$

in the set R with usual topology is

(A) open set
(B) closed set
(C) neither open nor closed
(D) both open and closed

20. The discrete space (X, D) is separable if and only if

(A) X is dense in itself
(B) X is countable
(C) X is infinite
(D) derived set of X is empty

21. A super market has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be served by the customer is 24 per hour. The probability that the customer is idle is

(A) 5/6
(B) 1/4
(C) 1/6
(D) 1/44

22. In a balanced transportation problem with m sources and n destinations the number of dual constraints is

(A) $m+n$
(B) $m+n-1$
(C) $m+n+1$
(D) mn

23. In the steady solution of the queuing model $(M/M/1) : (\infty/FCFS)$, the length of the system L_S and the length of the queue L_q are connected by the relation

(A) $L_S = \rho L_q$
(B) $L_q = \rho L_S$
(C) $L_S = \rho^2 L_q$
(D) $L_q = \rho^2 L_S$

24. For rigid body rotation at the neighborhood of a point p, which of the following is true?

(A) The spin tensor $V_{ij} = 0$
(B) The rate of deformation tensor $D_{ij} = 0$
(C) Both (A) and (B) are true
(D) $D_{ij} = V_{ij}$

25. Which of the following statement is false?

(A) Outer multiplication of tensors is commutative and associative.
(B) Inner multiplication of tensors is commutative and associative.
(C) A quantity which on inner multiplication by an arbitrary vector always gives a tensor.
(D) The outer product of two vectors is a tensor of order one and the converse is also true.

26. Which of the following statement is false?

(A) Covariant derivatives of fundamental tensor g^{ij} vanish identically.
(B) Covariant derivative of kroneckor delta does not vanish identically.
(C) Covariant derivative of a mixed tensor of rank two is a tensor of rank three.
(D) Covariant derivative of a covariant tensor of rank two is a tensor of rank three.

27. The equation of lines of flow relative to sphere is

(A) $\sin^2\theta = \dfrac{cr}{r^2 - a^2}$
(B) $\sin^2\theta = \dfrac{cr}{r^2 + a^2}$
(C) $\sin^2\theta = \dfrac{cr}{r^3 + a^3}$
(D) $\sin^2\theta = \dfrac{cr}{r^3 - a^3}$

28. "K.E. is lost by the collision of smooth elastic bodies whereas K.E. is gained by the internal explosion". This is the statement of

(A) Bertrand's theorem
(B) Kelvin's theorem
(C) Carnot's theorem
(D) Lagrange's theorem

29. Kinematics is concerned with the

(A) physical causes of the motion
(B) geometry of the motion
(C) conditions under which no motion is apparent
(D) None of the above

30. Which of the following is a Hilbert space?

(A) l^1
(B) l^4
(C) l^3
(D) l^2

31. The velocity vector $\vec{\boldsymbol{q}}$ in three-dimensional flow field for a liquid is given by $\vec{\boldsymbol{q}} = 2x\hat{\boldsymbol{i}} - y\hat{\boldsymbol{j}} + \lambda z\hat{\boldsymbol{k}}$. The value of λ is

(A) 0
(B) 1
(C) -1
(D) 2

32. The complex potential W for a two dimensional source of strength m placed at the origin is

(A) $W = -m\log z$
(B) $W = m\log z$
(C) $W = \dfrac{m}{z}$
(D) $W = -\dfrac{m}{z}$

33. Dimension of the dynamic viscosity μ is

(A) $ML^{-1}T^{-1}$
(B) L^2T^{-1}
(C) $ML^{-1}T^{-2}$
(D) ML^2T^{-1}

34. The velocity for a two dimensional liquid motion is given by $\vec{\boldsymbol{q}} = -x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$. The stream function is

(A) $\psi = \dfrac{x}{y}$
(B) $\psi = \dfrac{y}{x}$
(C) $\psi = -xy$
(D) $\psi = xy$

35. For Orthotropic material, the number of independent elastic constants is:

(A) 10
(B) 12
(C) 14
(D) 16

36. The relation between the radius vector r and the eccentric anomaly E for the elliptic motion is

(A) $r = e(1 - a\cos E)$
(B) $r = e(1 + a\cos E)$
(C) $r = a(1 + e\cos E)$
(D) $r = a(1 - e\cos E)$

37. Let p be "he is rich" and q be "he is happy" then which statement form will represent "he is neither happy nor rich"

(A) $(\sim (p \wedge q))$
(B) $\sim q \wedge \sim p$
(C) $\sim p \vee \sim q$
(D) $\sim p \rightarrow \sim q$

38. χ, the chromatic number of $K_{m,n}$ is

(A) $m + n$
(B) mn
(C) 2
(D) 1

39. If parallelogram law holds, then

(A) a normed space is a Hilbert space
(B) a normed space is both a Banach space as well as a Hilbert space
(C) a Banach space is a Hilbert space
(D) None of these holds

40. If a force

$$\vec{\boldsymbol{F}} = \left(\frac{ax - by}{x^2 + y^2}, \frac{ay + bx}{x^2 + y^2}, \frac{\lambda z}{x^2 + y^2}\right)$$

acting on a body is solenoidal, then the value of λ equals

(A) 2
(B) 1
(C) 0
(D) -1

41. The Klein 4-group is isomorphic to

(A) $Z_2 \times Z_2$
(B) Z_4
(C) Z_2
(D) None of these

42. Let G be a group of order 49. Then G is

(A) Abelian
(B) Cyclic
(C) Non-Abelian
(D) Centre of G has order 7

43. Of the fields Q, R, C and $GF(p)$ which of the following two are prime fields?

(A) Q and R
(B) R and C
(C) Q and $GF(p)$
(D) C and $GF(p)$

44. The function $f(x) = 2|x - 1| + 3|x - 2|$ is

(A) neither continuous nor differentiable at $x = 1, 2$
(B) continuous but not differentiable at $x = 1, 2$
(C) differentiable but not continuous at $x = 1, 2$
(D) differentiable at $x = 1, 2$

45. Which of the following statement is false?

(A) If $k(x,t)$ is symmetric, then the iterated kernels are also symmetric.

(B) $u(x) = 1$ is not a solution of the Fredholm integral equation

$$u(x) + \int_0^1 x(e^{xt} - 1)u(t)\,dt = e^x - x$$

(C) An integral equation is said to be singular, when one or both limits of integration become infinite.

(D) The eigen values of a symmetric kernel are real.

46. $z = a$ is an isolated singularity of $f(z)$ and $f(z) = \sum_{-\infty}^{\infty} a_n(z-a)^n$ is its Laurent expansion, then if $a_n = 0$ for $n \leq -1$; $z = a$ is

(A) an essential singularity

(B) a pole of order n

(C) a removable singularity

(D) None of the above

47. Consider $f(z) = z^{10} - 6z^7 + 3z^3 + 1$ and $|z| < 1$, then

(A) $f(z)$ has three zeros inside $|z| = 1$

(B) $f(z)$ has five zeros inside $|z| = 1$

(C) $f(z)$ has seven zeros inside $|z| = 1$

(D) $f(z)$ has ten zeros inside $|z| = 1$

48. The digit in the unit place in the number 7^{289} is

(A) 9

(B) 7

(C) 1

(D) 3

49. The number of primitive roots of 53 is

(A) 0

(B) 1

(C) 20

(D) 24

50. Euler's equation for the functional

$$\int_a^b F(y, y')\,dx$$

is

(A) $\frac{\partial F}{\partial y} - \frac{d}{dx}(\frac{\partial F}{\partial y'}) = 0$

(B) $\frac{\partial F}{\partial y} + \frac{d}{dx}(\frac{\partial F}{\partial y'}) = 0$

(C) $F - y'\frac{\partial F}{\partial y'} = C$, where C is a constant

(D) $F - y\frac{\partial F}{\partial y} = C$, where C is a constant

Section C (30 Marks)

Answer any 5 (five) from the following

1. Show that a finite group G is solvable if and only if G has a composition series whose factors are cyclic of prime order. (Marks : 6)

2. Show that the prime radical of a commutative ring R consists of all nilpotent elements of R. (Marks : 6)

3. Prove the equivalence of the following statements for the graph $G(p, q)$. (Marks : 6)

   (A) G is a tree.
   (B) G is connected and $p = q + 1$

4. Show that the series (Marks : 6)

$$\sum \frac{x}{(n^p + x^2 n^q)}$$

converges uniformly over any finite interval $[a, b]$ for $0 < p \leq 1$, $p + q > 2$.

5. Show that the resolvent kernel $\Gamma_{(x,t;\lambda)}$ of a non-homogeneous Volterra integral equation of 2nd kind (Marks : 6)

$$\phi(x) = f(x) + \lambda \int_0^x k(x,t)\phi(t)\,dt$$

can be expressed in the form

$$\Gamma_{(x,t;\lambda)} = k(x,t) + \lambda \int_t^x k(x,z)\Gamma_{(z,t;\lambda)}\,dz.$$

6. If $a > e$, then prove that the equation $e^z = az^n$ has n roots inside the circle $|z| = 1$. (Marks : 6)

7. Show that the equation of the plane curve of fixed perimeter and maximum area may be expressed in the form $(x - a)^2 + (y - b)^2 = \lambda^2$, where a and b are constants of integration and λ is Lagrange's multiplier. (Marks : 6)

8. Find the singular solution of $4(1 + z^3) = 9z^4pq$. (Marks : 6)

Space for Answers (Section C) : for Questions 1 to 8 (10 pages)

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