GU-RET 2016

Gauhati University Research Eligibility Test

Physics

Booklet Series : A

Invigilator's Name and Signature	Booklet No.	
	OMR Sheet No.	
	Roll No.	

Time : 2 hours 20 minutes **Total Marks : 80**

Number of Pages in this Booklet : **18**

Instructions for Candidates

1. Write your Roll No. and OMR Sheet No. in the boxes provided above.
2. This paper consists of two sections : **Section B** with 50 (fifty) multiple choice questions (MCQ) and **Section C** with 9 (nine) descriptive questions. Each MCQ has 4 (four) answers, out of which ONLY one is correct. You have to darken the circle (on the OMR Sheet) for the correct answer corresponding to the question given in this booklet.

   Example : (A) (B) (C) (D)

   where (C) is the correct answer. No marks will be given for markings made in this booklet. The descriptive questions in **Section C**, MUST be answered in the space provided in this booklet. **No extra pages will be provided in any case**.
3. Use a BLACK ball point pen in your OMR Sheet.
4. Read the instructions given inside this booklet before attempting to answer any questions.
5. DO NOT write your name, roll no, phone no, or anything, or put any marks anywhere in this booklet, otherwise your candidature will be disqualified.
6. If you are found to resort to any kind of unfair means such as carrying extra material other than pen, pencil, watch, eraser, and scale, or copying from somebody or from external material, your candidature will be disqualified.
7. Use of mobile phones, calculators, log tables or any other tables, wearable smart devices such as smart Android watches or objects of similar nature CAN NOT be used inside the examination hall.
8. At the end of the examination, you have to return this booklet and the OMR Sheet back to the invigilator.
9. There is no negative marks for incorrect answer.

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Section B (50 Marks)

1. A vector perpendicular to any vector on the plane $\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z} = 5$ is

   (A) $\boldsymbol{i} + \boldsymbol{j}$
   (B) $\boldsymbol{j} + \boldsymbol{k}$
   (C) $\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$
   (D) $\boldsymbol{i} + \boldsymbol{j} + 5\boldsymbol{k}$

2. The constraints represented by the equation $f(r_1, r_2, r_3, \ldots, t) = 0$ are said to be

   (A) Holonomic
   (B) Nonholonomic
   (C) Scleronomic
   (D) Bilateral

3. If Poisson bracket of any quantity with Hamiltonian vanishes, it indicates that quantity is

   (A) time independent
   (B) not dependent on time explicitly
   (C) truly a constant quantity
   (D) time dependent

4. Miller indices of one of the cube face is

   (A) (1 0 0)
   (B) (1 1 0)
   (C) (1 1 1)
   (D) (1 2 3)

5. The uncertainty in the momentum of a proton confined in a sphere of radius $\sim 10 - 15\,\mathrm{m}$ is at least of the order of

   (A) $10 - 22\,\mathrm{m\,kg/s}$
   (B) $10 - 21\,\mathrm{m\,kg/s}$
   (C) $10 - 20\,\mathrm{m\,kg/s}$
   (D) $10 - 19\,\mathrm{m\,kg/s}$

6. Bose-Einstein condensation can be achieved

   (A) in light bosons
   (B) in heavy bosons
   (C) in light fermions
   (D) in heavy fermions

7. Eigen values of the hermitian matrix

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

   (A) $-2, +2, 0$
   (B) $-2, +2, -2$
   (C) $+1, -1, +2$
   (D) $+1, -1, +1$

8. An e.m. wave incident on a conducting surface

   (A) is fully transmitted
   (B) is fully reflected
   (C) can penetrate up to certain depth
   (D) can penetrate most of its thickness

9. In a grand canonical ensemble

   (A) both energy and number of particles are constant
   (B) energy is constant but not the number of particles
   (C) number of particles is constant but not the energy
   (D) both the quantities are variable

10. A mathematical relation is given as $x^n \propto y^t$. Which one of the following type of graph is most suitable for the plot between x and y?

    (A) Linear
    (B) Semi log
    (C) Log-log
    (D) None of these

11. γ–ray cannot be used for structure determination of solid as

(A) its wavelength is much larger than inter-planar spacing of solids

(B) its wavelength is nearly of the same order as that of inter-planar spacing of solids

(C) its energy exceeds the binding energy of solids

(D) its wavelength is much shorter than inter-planar spacing of solids

12. The classical temperature dependence of electrical resistivity of a metal is

(A) $\rho \propto T^2$

(B) $\rho \propto 1/T^2$

(C) $\rho \propto T^{1/2}$

(D) $\rho \propto 1/T$

13. In case of a Rutherford scattering event, the differential scattering cross section is inversely proportional to

(A) $\sin\theta$

(B) $\sin^2\theta$

(C) $\sin^3\theta$

(D) $\sin^4\theta$

14. The Lagrangian of the Sun-Earth system is

(A) $\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{GmM}{r}$

(B) $-\frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - \frac{GmM}{r}$

(C) $\frac{1}{2}m\dot{r}^2 - \frac{GmM}{r}$

(D) $\frac{1}{2}mr^2\dot{\theta}^2 + \frac{GmM}{r}$

15. If a coordinate corresponding to rotation is cyclic, rotation of the system about a given axis remains invariant. Then which of the following quantity is conserved?

(A) Linear momentum

(B) Angular momentum

(C) Kinetic energy

(D) Potential energy

16. Jacobi identity for Poisson bracket

(A) $[X,[Y,H]] + [Y,[H,X] + [H,[X,Y]] = 0$

(B) $[X,[Y,H]] - [Y,[H,X] + [H,[X,Y]] = 0$

(C) $[X,[Y,H]] + [Y,[H,X] - [H,[X,Y]] = 0$

(D) $[X,[Y,H]] - [Y,[H,X] - [H,[X,Y]] = 0$

17. The de-Broglie wavelengths of a proton and an α particle are equal. The ratio of their velocities is

(A) 1 : 4

(B) 4 : 1

(C) 1 : 2

(D) 2 : 1

18. Zero point energy of harmonic oscillator is

(A) $\hbar\omega$

(B) $\frac{1}{2}\hbar\omega$

(C) $2\hbar\omega$

(D) $\frac{1}{4}\hbar\omega$

19. The value of $[L_x, L_y]$ and $[L^2, L_y]$ is

(A) $i\hbar L_z$ and 1

(B) $-i\hbar L_z$ and 1

(C) $-i\hbar L_z$ and 0

(D) $i\hbar L_z$ and 0

20. The parities of the wave functions (i) $\cos(kx)$, and (ii) $\tanh(kx)$ are

(A) (i) odd, (ii) odd

(B) (i) even, (ii) even

(C) (i) odd, (ii) even

(D) (i) even, (ii) odd

21. A system in a normalized state $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$ with $|\alpha_1\rangle$and $|\alpha_2\rangle$ representing two different eigen states of the system, requires that the constants c_1 and c_2 must satisfy the condition

(A) $|c_1| \cdot |c_2| = 1$

(B) $|c_1| + |c_2| = 1$

(C) $(|c_1| \cdot |c_2|)^2 = 1$

(D) $|c_1|^2 + |c_2|^2 = 1$

22. The de-Broglie wavelength of a particles in thermal equilibrium at temperature T is

(A) $\dfrac{h}{\sqrt{2mkT}}$

(B) $\dfrac{\hbar}{\sqrt{2mkT}}$

(C) $\dfrac{\hbar}{\sqrt{mkT}}$

(D) $\dfrac{h}{\sqrt{2kT}}$

23. Two masses m and $3m$ are attached to two ends of a massless spring with force constant k. If $m = 100\,\text{g}$ and $k = 0.3\,\text{N/m}$, the natural angular frequency of oscillation is

(A) 2 Hz

(B) 0.2 Hz

(C) 2 kHz

(D) 0.2 kHz

24. A radioactive nucleus of initial mass no. A and atomic no. Z, emits three α particles and two positrons. The ratio of no. of neutrons to that of protons in the final state is

(A) $\dfrac{A-Z-8}{Z-4}$

(B) $\dfrac{A-Z-4}{Z-8}$

(C) $\dfrac{A-Z-12}{Z-4}$

(D) $\dfrac{A-Z-4}{Z-2}$

25. In β^+ decay process, the following changes takes place inside the nucleus

(A) ${}^A_Z X \to {}^A_{Z-1} Y + e^+ + \nu$

(B) ${}^A_Z X \to {}^A_{Z+1} Y + e^- + \bar{\nu}$

(C) ${}^A_Z X \to {}^A_Z Y + e^- + \nu$

(D) ${}^A_Z X \to {}^A_Z Y + e^- + \bar{\nu}$

26. Cadmium is used as control rod in fission reactors as

(A) it has large neutrino cross-section

(B) it has large neutron cross-section

(C) it has large proton cross-section

(D) None of the above

27. Experimentally it is found that at the core of a nucleus, the nuclear density remains same. This observation suggests that at very short distance, nucleon-nucleon interactions is

(A) attractive

(B) repulsive

(C) spin dependent

(D) non-central in nature

28. Which of the following high energy processes is allowed by conservation laws?

(A) $p \to n + e^+$

(B) $p \to e^+ + \nu_e$

(C) $p \to \pi^+ + \gamma$

(D) $\bar{p} + n \to \pi^- + \pi^0$

29. Plane electromagnetic wave of frequency 30 MHz travels in free space along x-direction. The electric field component of the wave at a particular point of space and time is $E = 6\,\text{Vm}^{-1}$ along y-direction. Its magnetic field component B at this point would be

(A) $2 \times 10^{-8}\,\text{T}$ along z direction

(B) $6 \times 10^{-8}\,\text{T}$ along x direction

(C) $2 \times 10^{-8}\,\text{T}$ along y direction

(D) $6 \times 10^{-8}\,\text{T}$ along z direction

30. The electric field $\boldsymbol{E}$ at the centre of a uniformly charged conductor of radius r is

(A) $\dfrac{q}{4\pi\varepsilon_0 r^3}$

(B) $\dfrac{q}{4\pi\varepsilon_0 r^2}$

(C) zero

(D) $\dfrac{q}{4\pi\varepsilon_0 r}$

31. The energy per unit time, per unit area transported by an e.m. field is expressed as

(A) $\boldsymbol{S} = \dfrac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B})$

(B) $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{B}$

(C) $\boldsymbol{S} = \mu_0(\boldsymbol{E} \times \boldsymbol{B})$

(D) $\boldsymbol{S} = \dfrac{1}{\varepsilon_0}(\boldsymbol{E} \times \boldsymbol{B})$

32. In an electric dipole, electric field intensity

(A) is $\propto \frac{1}{r^2}$

(B) is $\propto \frac{1}{r^3}$

(C) is $\propto \frac{1}{r}$

(D) is $\propto \frac{1}{r^4}$

33. The effective mass of an electron in a semiconductor

(A) can never be positive

(B) can never be negative

(C) can never be positive or negative

(D) depends on its spin

34. A superconducting ring is cooled in the presence of a magnetic field below its critical temperature (T_C). The total magnetic flux linked that passes through the ring is

(A) zero

(B) $n\frac{h}{2e}$

(C) $n\frac{h}{4\pi e}$

(D) $n\frac{e^2}{hc}$

35. The eigen values of a matrix are $i, -2i$ and $3i$. The matrix is

(A) unitary

(B) anti-unitary

(C) Hermitian

(D) anti-Hermitian

36. A unit vector normal to the surface $x^3 - xyz = 1$ at the point $(1, 1, 1)$

(A) $\frac{\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}}{\sqrt{3}}$

(B) $\frac{2\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}}{\sqrt{6}}$

(C) $\frac{2\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}}{\sqrt{6}}$

(D) $\frac{2\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}}}{\sqrt{5}}$

37. If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of the matrix

$$\begin{bmatrix} -2 & -9 & 5 \\ -5 & -10 & 7 \\ -9 & -21 & 14 \end{bmatrix},$$

then $\lambda_1 + \lambda_2 + \lambda_3$ is equal to

(A) -16

(B) 2

(C) -6

(D) -14

38. The solutions of the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are

(A) family of circles with different radii

(B) circles with different centres

(C) straight line with different slopes

(D) straight lines with different intercepts on the y-axis

39. What should be the clock frequency of a 6-bit A/D converter, 32 μs?

(A) 1 MHz

(B) 2 MHz

(C) 0.5 MHz

(D) 4 MHz

40. The lifetime for an ordinary excited state of an atom is 10^{-8} sec. The natural half-intensity width of a spectral line due to transition from this state to the ground state is

(A) $10^7\ \mathrm{s}^{-1}$

(B) $10^8\ \mathrm{s}^{-1}$

(C) $6.28 \times 10^8\ \mathrm{s}^{-1}$

(D) $6.4 \times 10^7\ \mathrm{s}^{-1}$

41. The FWHM of a diode laser of wavelength 800 nm is 1 nm. Its temporal coherence length is

(A) 640 μm

(B) 640 nm

(C) 800 μm

(D) 800 nm

42. Which of the following statements regarding Raman scattering is not correct?

(A) Raman scattering is an inelastic scattering

(B) Stokes' radiation is usually more intense than anti-Stokes' one

(C) Symmetric vibrations give rise to weak while non-symmetric vibrations give rise to intense Raman lines

(D) If a molecule has a centre of symmetry, Raman active vibrations are infrared inactive and vice versa

43. Hot bands in the vibrational spectra are given by the transition

(A) $\nu = 0 \rightarrow \nu = 1$

(B) $\nu = 0 \rightarrow \nu = 2$

(C) $\nu = 0 \rightarrow \nu = 3$

(D) $\nu = 1 \rightarrow \nu = 2$

44. A laser resonator is formed by two mirrors facing each other, placed at a distance of 1 m apart, and containing an active medium of index of refraction 1.5. The spacing between adjacent longitudinal modes will be

(A) 10^8 Hz

(B) 4×10^8 Hz

(C) 4.4×10^{-10} Hz

(D) 10^{-8} Hz

45. Which one of the following is true for electronic devices? Input impedance of

(A) Transistor > FET > MOSFET

(B) FET > Transistor > MOSFET

(C) MOSFET < Transistor < FET

(D) MOSFET > FET > Transistor

46. The length of an antenna for efficient reception and transmission of message of 1 MHz is

(A) 75 m

(B) 100 m

(C) 150 m

(D) None of the above

47. If the electric field of an e.m. wave propagating in free space is given by $E(z,t) = E_0 e^{i(kx-\omega t)}\hat{\boldsymbol{x}}$, which of the following represents the magnetic field?

(A) $B(x,t) = \frac{1}{c} E_0 e^{i(kz-\omega t)}\hat{\boldsymbol{x}}$

(B) $B(y,t) = \frac{1}{c} E_0 e^{i(kz-\omega t)}\hat{\boldsymbol{y}}$

(C) $B(z,t) = \frac{1}{c} E_0 e^{i(kz-\omega t)}\hat{\boldsymbol{y}}$

(D) $B(z,t) = \frac{1}{c} E_0 e^{i(kz-\omega t)}\hat{\boldsymbol{x}}$

48. Which of the following statements is true for an e.m. wave propagating in a conductor?

(A) The magnetic field lags behind the electric field.

(B) The imaginary part of the wave vector is related to the refractive index.

(C) The electric and magnetic fields are not transverse.

(D) The ratio of the amplitudes of the electric and magnetic fields is related to the frequency

49. Which one of the following is true about the spectra of a FM modulated signal?

(A) When modulation index of FM is small, the carrier disappears

(B) The carrier of FM disappears when the modulation index is large

(C) The number of sidebands depend only on the frequency of the modulating signal

(D) The number of sidebands depends on the amplitude and frequency of the modulating signal

50. Which one is true for OPAMP?

(A) For non-inverting amplifier, input impedance increase than without feedback

(B) For non-inverting amplifier, output impedance is greater than without feedback

(C) For inverting amplifier, the input impedance becomes very large always

(D) For inverting amplifier, the input impedances and output impedance are like the ideal

Section C (30 Marks)

Answer any 5 (five) from the following

1. Show that $f(\xi) = \xi e^{\xi^2/2}$ is an eigen function of the linear harmonic oscillator equation

$$\frac{d^2}{d\xi^2}\psi(\xi) + (\lambda - \xi^2)\psi(\xi) = 0$$

corresponding to the eigen value $\lambda = -3$. (Marks : 6)

2. What is the importance of a partition function in statistical mechanics? Calculate the canonical partition function for a quantum harmonic oscillator. The entropy of a grand canonical system is expressed in terms of partition function Z as $S = k_B[\beta \langle E \rangle + \beta\mu \langle N \rangle + \ln Z]$, with $\langle E \rangle$ and $\langle N \rangle$ average energy and number of particles and $\beta = 1/(k_B T)$. Interpret the chemical potential μ, from this relation. If the pressure of the system is $\langle P \rangle = \beta^{-1}\partial \ln Z/\partial V$, show that $\langle P \rangle \approx -\partial \langle E \rangle / \partial V$ for a canonical system without thermal coupling. (Marks : $1 + 2 + 3 = 6$)

3. Based on the Franck Condon principle, explain the variation of intensity in electronic spectral bands for the following cases (Marks : 6)

   (A) The upper electronic state has the same internuclear distance as the lower one.

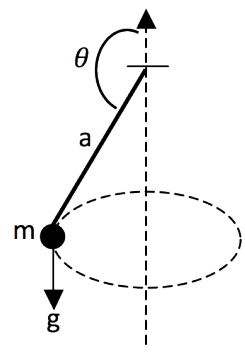
   (B) The upper electronic state has the internuclear distance a little bit less than in the lower one.

   (C) The upper electronic state has the internuclear distance a little greater than in the lower one.

4. A particle of mass m is attached to a fixed point by a weightless inextensible string of length a. It is rotating under gravity as shown in the figure below. The Lagrangian of the particle is

$$L(\theta, \phi) = \frac{1}{2}ma^2(\theta^2 + \sin^2\theta\,\phi^2) - mga\,\cos\theta,$$

where θ and ϕ are the polar angles. Calculate the Hamiltonian of the particle. (Marks : 6)



5. Establish, which of the mirror nuclei ${}_{52}\mathrm{Te}^{127}$ and ${}_{53}\mathrm{I}^{127}$ decays into the other. Which kind of decay would it be? Justify your answer. (Marks : 6)

6. For the harmonic oscillator $[V(x) = (1/2)kx^2]$, the allowed energies are

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, \qquad n = 0, 1, 2, \ldots,$$

where $\omega = \sqrt{k/m}$ is the classical frequency. Assume that the spring constant increases slightly: $k \to (1+\epsilon)k$, calculate the first-order correction in the energy, using perturbation theory. (Marks : 6)

7. Evaluate the following integral using the method of complex variables

$$\oint_C \frac{z^2}{(z-1)^2(z+2)}\, dz,$$

where C is the circle with $|z| = 3$. (Marks : 6)

8. It is required to analyze an analog signal of 1 kHz by digitizing the signal. What are the things to be kept in mind to minimize the error in frequency and amplitude? (Marks : 6)

9. Sodium crystallizes in bcc structure. If the radius of the sodium atom is 1.75 nm, calculate the spacing between (1 1 1) planes. (Marks : 6)

Space for Answers (Section C) : for Questions 1 to 9 (9 pages)

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